

Electrostatic ion perturbations in unmagnetized plasma shear flow

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Abstract

The electrostatic perturbations in an unmagnetized, non-isothermal ($T_i \ll T_e$) electron-ion plasma shear flow are considered. New physical effects, arising due to the non-normality of linear dynamics, are described. It is shown that the velocity shear induces the extraction of the mean flow energy by the acoustic perturbations (ion-sound waves). The influence of the medium dispersion, rising due to the violation of quasineutrality for perturbations, is examined. It is shown that in the course of the evolution ion-sound waves turn into ion plasma oscillations. New class of nonperiodic, electrostatic perturbations (with vortical motion of ion component), characterized by the intense energy

exchange with the mean flow, is also described.

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Classical stability theory of continuous media motion (normal mode approach) has been victorious in explaining how different kinds of shear flows become unstable. However, in some, quite simple and important kinds of *parallel free shear flows* (e.g., plane Couette and Poiseuille, or pipe Poiseuille flows) the approach has serious problems, evoked by the not self-adjoint character of the governing equations [1–3]. That is why the predictions of the traditional stability approach fail to match the results of most experiments with these kinds of flows [3].

An alternative approach to the problem is that due to Kelvin [4], which implicates the change of independent variables from a laboratory to a moving frame and study of the non-exponential temporal evolution of *spatial Fourier harmonics* (SFH). The method is operative for any mean velocity profiles but it is most manageable to ones that are linear, or peacewise linear [1,5]. Effectiveness of the method was repeatedly proved, when it helped to obtain the bunch of unlooked-for results on the dynamics of the perturbations in hydrodynamical [6–12] and hydromagnetic [13–15] shear flows.

In [12] authors have considered the evolution of two-dimensional (2D) SFH in a compressible, plane hydrodynamical Couette flow. Their analysis, which involved the nonmodal approach, brought them to the discovery of the new mechanism of the energy exchange between the mean flow and sound-type perturbations. In particular, it was shown that energy of SFH may grow linearly in time—perturbations extract the energy from the mean shear flow. This process appears to be quite universal and one should expect that it may be also influential in the wide variety of continuous media with the analogous kinematics.

In this *report* we shall examine the case of electron-ion plasma shear flow and show that the process of velocity shear induced energy transfer from the mean flow to the collective modes exists and can be quite efficient in this case too. Moreover, as we shall see, the peculiarity of the plasma state of the medium plays its distinctive role and leads to the whole group of interesting new effects.

As it is well-known, in the collisionless unmagnetized plasma with $T_i \ll T_e$ (where T_e and T_i are electron and ion temperatures respectively) exists the weakly damped low-frequency,

longitudinal, and therefore electrostatic, ion mode. When its wave-length $\lambda \sim 1/k$ (k is wave number) greatly exceeds electron Debye length ($\lambda \gg \lambda_{De} \equiv (T_e/4\pi e^2 n_0)^{1/2}$, then this low-frequency mode represents the *ion-sound wave* with constant phase velocity (‘ion acoustic’ speed): $C_s \equiv (T_e/m_i)^{1/2}$. However, the velocity shear induces the “linear drift” of SFH (the process well-acknowledged in the literature [6,12,13] cultivating the nonmodal approach) which, mathematically, is exposed in the temporal variation of the wave number vector $\mathbf{k}(t)$. It means that the influence of the *dispersion* of ion mode, arising as the result of the violation of quasineutrality for the perturbations, which, in other words, is related to the finiteness of $k\lambda_{De}$, should be taken into due account. The reason is simple: if initially $|k\lambda_{De}| \ll 1$ the linear drift process will, eventually, transfer the SFH to the region of $\mathbf{k} - space$, where the latter condition does not hold. In physical terms it means that that under certain circumstances the ion sound waves, drawing energy from the mean shear flow, subsequently turn into the ion plasma oscillations. The latter collective mode is weakly damped if $\lambda \gg \lambda_{Di} \equiv (T_i/4\pi e^2 n_0)^{1/2}$.

Velocity shear of the mean flow induces, also, excitation of the completely new kind of non-periodic, electrostatic perturbations with vortical motion of the plasma ion component. These perturbations are able to the effective exchange of their energy with the mean flow and under certain conditions may play the *dominant* role in the behavior of the plasma flow.

At the end of this report we shall point out on the possible relevance of above noted remarkable processes to the problems of excitation of electrostatic waves, resonant acceleration of particles and onset of a turbulence in non magnetized electron-ion plasmas.

Let us consider electrostatic oscillation modes in the shear flow of electron-ion nonmagnetized plasma. The mean velocity $\mathbf{U}_0 \equiv (Ay, 0, 0)$ is directed along the X axis and has a linear shear along the Y axis. Without the loss of generality we can assume that the constant parameter A is positive. Equilibrium electric field \mathbf{E} is zero. Taking approval of the electrostatic nature of oscillations we can write the electric field perturbation as the gradient of the electric potential: $\mathbf{E} = -\nabla\phi$. The plasma in its equilibrium state is assumed to be homogeneous and quasineutral—equilibrium number density of electrons n_{e0} is equal to the one of ions n_{i0} and is constant: $n_{e0} = n_{i0} = n_0 = const$.

When the plasma is properly rarefied, ion and electron components interact weakly with each other and may, thus, have very different temperatures. In particular, we shall assume that temperature of electrons is homogeneous $T_e = \text{const}$ and for the sake of simplicity take that $T_i = 0$. The effects, which may be evoked by the finiteness of T_i will be discussed briefly at the end of the report.

The usual approximation for electrostatic low-frequency waves implies that the electron number density can be described by the Boltzmann distribution [16]:

$$n_e = n_0 \exp(e\phi/T_e) \approx n_0(1 + e\phi/T_e), \quad (1)$$

Let us, now, decompose the instantaneous values of all physical variables onto their mean and perturbed components: $\mathbf{V}_i = \mathbf{U}_0 + \mathbf{u}$, $n_{e,i} = n_0 + n'_{e,i}$. The basic system of linearized equations for ions, describing the evolution of small-scale, 3D perturbations in this flow, takes the form:

$$[\partial_t + Ay\partial_z]n'_i + n_0[\partial_x u_x + \partial_y u_y + \partial_z u_z] = 0, \quad (2)$$

$$[\partial_t + Ay\partial_x]\mathbf{u} + (\mathbf{u}, \nabla)\mathbf{U}_0 = -\frac{e}{m_i}\nabla\phi, \quad (3)$$

$$[\partial_x^2 + \partial_y^2 + \partial_z^2]\phi = 4\pi e(n'_e - n'_i). \quad (4)$$

where $(\mathbf{u}, \nabla)\mathbf{U}_0$ is a vector, which has only one, x-th, nonzero component equal to Au_y .

Let us make the following substitution of variables: $x_1 = x - Ayt$; $y_1 = y$; $z_1 = z$; $t_1 = t$ and perform the Fourier analysis of (2–4), expanding unknown functions with respect to spatial coordinates:

$$F = \int dk_{x_1} dk_{y_1} dk_{z_1} \hat{F}(k_{x_1}, k_{y_1}, k_{z_1}, t_1) \exp[i(k_{x_1}x_1 + k_{y_1}y_1 + k_{z_1}z_1)], \quad (5)$$

where under F we imply all physical variables appearing in the problem. Instead of equations (2) and (3) we get (for details, see [9,13]) the following set of first order, ordinary differential equations (ODE's) for SFH of these variables:

$$N_i^{(1)} = v_x + \beta(\tau)v_y + \gamma v_z, \quad (6)$$

$$v_x^{(1)} = -Rv_y - \Phi, \quad (7a)$$

$$v_y^{(1)} = -\beta(\tau)\Phi, \quad (7b)$$

$$v_z^{(1)} = -\gamma\Phi; \quad (7c)$$

complemented by the algebraic relations, arising from (1) and (4):

$$N_e = \Phi, \quad (8)$$

$$N_i = [1 + \xi^2 \mathcal{K}^2] N_e, \quad (9)$$

where, hereafter, $F^{(n)}$ will denote the n -th order time derivative of F .

In (6–9) we have introduced the following dimensionless notations: $R \equiv A/C_s k_{x_1}$, $\tau \equiv C_s k_{x_1} t_1$, $\beta_0 \equiv k_{y_1}/k_{x_1}$, $\beta \equiv \beta_0 - R\tau$, $\gamma \equiv k_{z_1}/k_{x_1}$, $\mathbf{v} \equiv \hat{\mathbf{u}}/C_s$, $\xi \equiv k_{x_1} C_s/\omega_p = \lambda_{De} k_{x_1}$, $\Phi \equiv i e \hat{\phi}/T_e$, $N_{i,e} \equiv i \hat{n}'_{i,e}/n_0$. Here $\omega_p \equiv (4\pi n_0 e^2/m_i)^{1/2}$ is the ion plasma frequency. Note that $\mathcal{K} \equiv |k(t_1)|/k_{x_1} = (1 + \beta^2(\tau) + \gamma^2)^{1/2}$ is the modulus of the dimensionless and time-dependent wave number vector of SFH.

Note that the time-dependence of $\beta(\tau)$ and $\mathcal{K}(\tau)$ is provoked by the temporal variability of the SFH wave number component along the flow shear:

$$k_y(\tau) \equiv k_{y_1} - R k_{x_1} \tau = k_{x_1} \beta(\tau). \quad (10)$$

This process of the "linear drift" in the \mathbf{k} -space [13] has the crucial role for the temporal evolution of SFH.

The total energy of perturbations consists of the kinetic energy of ions, compressional energy associated with electrons (ions does not contribute, because their temperature is zero) and the energy of electric field. The phenomenological expression for the spectral density of the total energy is equal to:

$$\mathcal{E} \equiv \frac{1}{2} \left\{ |v_x|^2 + |v_y|^2 + |v_z|^2 + [1 + \xi^2 \mathcal{K}^2] |\Phi|^2 \right\}. \quad (11)$$

The first order ODE for $\mathcal{E}(\tau)$, derived from (6–9) is:

$$\mathcal{E}^{(1)} = -R \left[v_x v_y + 2 \xi^2 \beta(\tau) \Phi^2 \right], \quad (12)$$

and in the "no-shear" ($R = 0$) limit it is the conserved quantity.

It is easy to verify that variables appearing in Eq. (6–9) obey the following remarkable *algebraic* relation:

$$(1 + \gamma^2)v_y - \beta(\tau)(v_x + \gamma v_z) = RN_i + \text{const.} \quad (13)$$

By means of this relation, taking a derivative of eq. (6) and re-arranging terms, we can get the following explicit second order ODE for the function $\Psi \equiv N_i/\mathcal{K}$:

$$\Psi^{(2)} + \Omega^2(\tau)\Psi = \text{const} \times f(\tau), \quad (14)$$

where

$$\Omega^2(\tau) \equiv \frac{\mathcal{K}^2}{1 + \xi^2 \mathcal{K}^2} + \frac{3R^2(1 + \gamma^2)}{\mathcal{K}^4}, \quad (15a)$$

$$f(\tau) \equiv -2R/\mathcal{K}^3. \quad (15b)$$

It should be noted that the analogous, but more simple, kind of inhomogeneous second-order ODE was derived and analyzed in [12]. The general solution of (14) is the sum of its *special* solution and the *general* solution of the corresponding homogeneous ($\text{const} = 0$) equation: $\Psi = \Psi_h + \Psi_s$. When $\Omega(\tau)$ depends on τ adiabatically, implying [12,17]:

$$|\Omega(\tau)^{(1)}| \ll \Omega^2(\tau), \quad (16)$$

then the homogeneous equation can be solved approximately.

For the flows with $R \ll 1$ the condition (16) holds for the wide range of possible values of $|\beta(\tau)|$. In other words, since the temporal variability of $|\beta(\tau)|$ is determined by the "linear drift" of SFH, (16) is valid at all stages of the evolution of the SFH. When the condition (16) holds, the approximate expression for Ψ_h may be written in the following way:

$$\Psi_h(\tau) \approx \frac{C}{\sqrt{\Omega(\tau)}} \exp[i(\varphi(\tau) + \varphi_0)], \quad (17)$$

where $\varphi(\tau) = \int_0^\tau \Omega(\tau') d\tau'$.

As regards the special solution of inhomogeneous equation (14), it can also be derived owed to the smallness of the R parameter. In particular, the solution may be expressed by

the following series [18]:

$$\Psi_s(\tau) = \text{const} \times \sum_{n=0}^{\infty} R^{2n} y_n(\tau), \quad (18a)$$

$$y_0(\tau) = f(\tau)/\Omega^2(\tau), \quad (18b)$$

$$y_n(\tau) = -\frac{1}{\Omega^2(\tau)} \frac{\partial^2 y_{n-1}}{\partial \beta^2}. \quad (18c)$$

Since $R \ll 1$, the terms with higher powers of R are negligible and the *full* approximate solution of the inhomogeneous equation (14) may be written explicitly as:

$$\Psi = \Psi_h + \Psi_s \approx \frac{C}{\sqrt{\Omega(\tau)}} \exp[i(\varphi(\tau) + \varphi_0)] + \frac{\text{const} \times f(\tau)}{\Omega^2(\tau)}. \quad (19)$$

When $C/\text{const} \ll 1$ the SFH may be treated as mainly incompressible and vortical perturbation, while when $C/\text{const} \gg 1$ it is mainly of the sound-type.

Having in hands the solution of (14) we are, certainly, able to find all other variables of the problem. They may be calculated by Ψ and $\Psi^{(1)}$ in the following way:

$$N_i = \mathcal{K}\Psi, \quad (20)$$

$$N_e = \mathcal{K}\Psi/(1 + \xi^2 \mathcal{K}^2), \quad (21)$$

$$v_y = \frac{1}{\mathcal{K}^3} \left[\beta \mathcal{K}^2 \Psi^{(1)} + R(1 + \gamma^2) \Psi + \text{const} \times \mathcal{K} \right], \quad (22)$$

$$v_x + \gamma v_z = \frac{1}{\mathcal{K}^3} \left[(1 + \gamma^2) \mathcal{K}^2 \Psi^{(1)} - R\beta(1 + \gamma^2 + \mathcal{K}^2) \Psi - \text{const} \times \beta \mathcal{K} \right], \quad (23)$$

Note that (20–23) are exact expressions, valid for *arbitrary* values of the shear parameter R . For large enough values of R , when adiabatic solution (19) is no longer valid, Eqs. (20–23) may still be used for reproduction of the variables through Ψ and $\Psi^{(1)}$, obtained, this time, by the direct numerical solution of equation (14).

Below we shall focus our attention on the behavior of 2D perturbations in the XOY plane ($\gamma = 0$), when the shear parameter $R \ll 1$. This case admits simple analytical examination and exposes soundly the qualitative novelty of the problem. Using expression (11) for the spectral energy density $\mathcal{E}(\tau)$ and (20–23), we get for $\mathcal{E}(\tau)$ the following simple expression:

$$\mathcal{E}(\tau) \simeq \frac{1}{2} \left[C^2 \Omega(\tau) + \left(\frac{\text{const}}{\mathcal{K}(\tau)} \right)^2 \right]. \quad (24)$$

The spatial characteristics of the SFH (k_{x1} , $k_y(\tau)$) and the value of the shear parameter R manage the evolution of the frequency of oscillations and the actual intensity of the energy exchange between the SFH and the background flow. In particular, temporal variability of these processes are essentially induced by the "linear drift" of SFH in the \mathbf{k} -space [13,12].

For the sound-type ($C \neq 0$, $const = 0$) perturbations, as it is evident from (15a), the frequency of oscillations varies with the variation of $\mathcal{K}(\tau)$. Originally, at moderate values of $\mathcal{K}(\tau)$, due to the smallness of ξ , the oscillation mode may be treated as the ion-sound wave ($\Omega(\tau) \sim \mathcal{K}^2(\tau)$). Afterwards, when $\mathcal{K}(\tau)$ reaches large enough values, the dispersive influence of the denominator in the first term of (15a) becomes more and more imperative and when $\xi^2 \mathcal{K}^2(\tau) \gg 1$ the frequency $\Omega(\tau)$ already exhibits ion plasma oscillations. Following, according to (24), the evolution of energy of this mode ($\mathcal{E}(\tau) \sim \Omega(\tau)$), we find that initially, for $\beta_0 > 0$, at $0 < \tau < \tau_* \equiv |\beta_0|/R$, the energy decreases and reaches its minimum at $\tau = \tau_*$. A while later, it begins to increase at $\tau_* < \tau < \infty$, when the SFH "emerges" into the area of \mathbf{k} -space in which $k_y(\tau)k_{x1} < 0$ (the "growth area" for the sound-type perturbations [12]). If the SFH is in the "growth area" from the beginning ($\beta_0 < 0$), its energy increases monotonically. When $\xi \mathcal{K}(\tau) \geq 1$ the rate of the energy increase becomes less and less and the energy asymptotically tends to the constant value.

When $C = 0$ and $const \neq 0$ the SFH may be treated as mainly incompressible and vortical perturbations. In this case $\Psi \simeq const \times f(\tau)/\Omega^2(\tau)$, while $v_y \simeq const/\mathcal{K}^2(\tau)$ and $v_x \simeq -const \times \beta/\mathcal{K}^2(\tau)$. The spectral energy of SFH varies as $\mathcal{E}(\tau) \simeq \mathcal{K}^{-2}(\tau)$ and reduces to the well-known expression, describing the "transient" growth of the energy of SFH [6,7,13]. Transient increase of the energy takes place if initially $k_{y1}/k_{x1} > 0$ ($\beta_0 > 0$) and occurs nearby the $\tau_* \equiv \beta_0/R$ moment of time, when $\beta(\tau)$ tends to zero and $\mathcal{K}(\tau)$ attains its minimum value. Such is the behavior of 2D vortical perturbations. One should expect that the evolution of three-dimensional (3D) vortical perturbations should be similar to the behavior of the analogous structures in incompressible, inviscid fluids, extensively studied in [9]. As it was shown in [9], the energy of 3D vortical perturbations grows also nonexponentially, but unlike transiently growing 2D perturbations, the energy of 3D perturbations saturates,

attaining in asymptotics some constant value.

Above noted similarity of the behavior of vortical perturbations with the same process in usual fluids [12], holds only in the low- R ($R \ll 1$) range. For larger values of the R parameter ($R \simeq 1$) one should expect notable differences between the behavior of vortical perturbations in neutral fluids and electron-ion plasma.

Certainly, in the general case ($C \simeq \text{const}$), the "vortical" and the "sound-type" evolution of perturbations are superimposed on one another.

Thus, we see that the dispersive nature of plasma, arising due to the violation of the quasineutrality for density perturbations, leads to the smooth transition of ion-sound waves, amplified by the process of a shear flow energy extraction, into the ion plasma oscillations. As regards to the another type of dispersion properties of ion oscillations, related to the finiteness of *ion* temperature (and, for simplicity, neglected in the above consideration) they should lead to the transfer of the energy to the shorter wave length region. The characteristic phase velocity of perturbations in the course of the "linear drift" of SFH, tends to ion-thermal velocity. Due to the resonant interaction with ions these perturbations will be damped (Landau damping) transferring their energy to the particles and accelerating them. However, we should notice that such channel of dissipation and collisionless heating of plasma ion component is relevant for perturbations with characteristic time scale smaller, than the time scale for electron-ion collisions. The energy can also become one of the main sources for the onset of plasma turbulence in such flows.

Surely, the problem under consideration is quite complex. The comprehensive analysis of all possible regimes is beyond the scope of this report and will be published elsewhere.

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REFERENCES

- [1] W.O. Criminale and P.G. Drazin *Stud. Appl. Maths.*, **83**, 123 (1990).
- [2] S.C. Reddy and D.S. Henningson, *J. Fluid Mech.* **252**, 209 (1993).
- [3] L.N. Trefethen, A.E. Trefethen, S.C. Reddy, and T.A. Driscoll, *Science* **261**, 578 (1993).
- [4] Lord Kelvin (W. Thomson), *Phil. Mag.* **24**, Ser. **5**, 188 (1887).
- [5] W.O. Criminale, T.L. Jackson, and D.G. Lasseigne, *J. Fluid. Mech.* **294**, 283 (1995).
- [6] P. Marcus, W.H. Press, *J.Fluid Mech.* **79**, 525 (1977).
- [7] A.D.D. Craik, W.O.Criminale, *Pros. R. Soc. Lond.* **A406**, 13 (1986).
- [8] J.G. Lominadze, G.D. Chagelishvili, and R.A. Chanishvili, *Pis'ma Astron. Zh.* **14**, 856 (1988) [*Sov. Astron. Lett.* **14**, 364 (1988)].
- [9] G.D. Chagelishvili, R.G. Chanishvili, J.G. Lominadze, and I.N. Segal, *Proceedings of the fourth International Conference on Plasma Physics and Controlled Nuclear Fusion*, held in Toki, Japan 17-20 November 1992 (ESA SP-351, 1993).
- [10] K.M. Butler and B.F. Farrell, *Phys. Fluids A* **4**, 1637 (1992).
- [11] L.H. Gustavsson, *J.Fluid Mech.* **224**, 241 (1991).
- [12] G.D. Chagelishvili, A.D. Rogava, and I.N. Segal, *Phys. Rev. (E)* **50**, 4283 (1994).
- [13] G.D. Chagelishvili, T.S. Christov, R. G. Chanishvili, and J.G. Lominadze, *Phys. Rev. (E)* **47**, 366 (1993).
- [14] S.A. Balbus, and J.H. Hawley, *Ap.J.* **400**, 610 (1992).
- [15] S.H. Lubow, and H.C. Spruit, *Ap.J.* **445**, 337 (1995).
- [16] R.J. Goldston and P.H. Rutherford *Introduction to Plasma Physics* (Institute of Physics Publishing, Bristol 1995).

- [17] Zel'dovich Ya. B. and Mishkis A. D. *Elementi Prikladnoi Matematiki* (Nauka, Moscow 1972) (in russian).
- [18] Magnus K. 1976, *Schwingungen* (B. G. Teubner, Stuttgart).